Modelovanie regionálneho tiažového poľa z meraní družicovej misie GOCE

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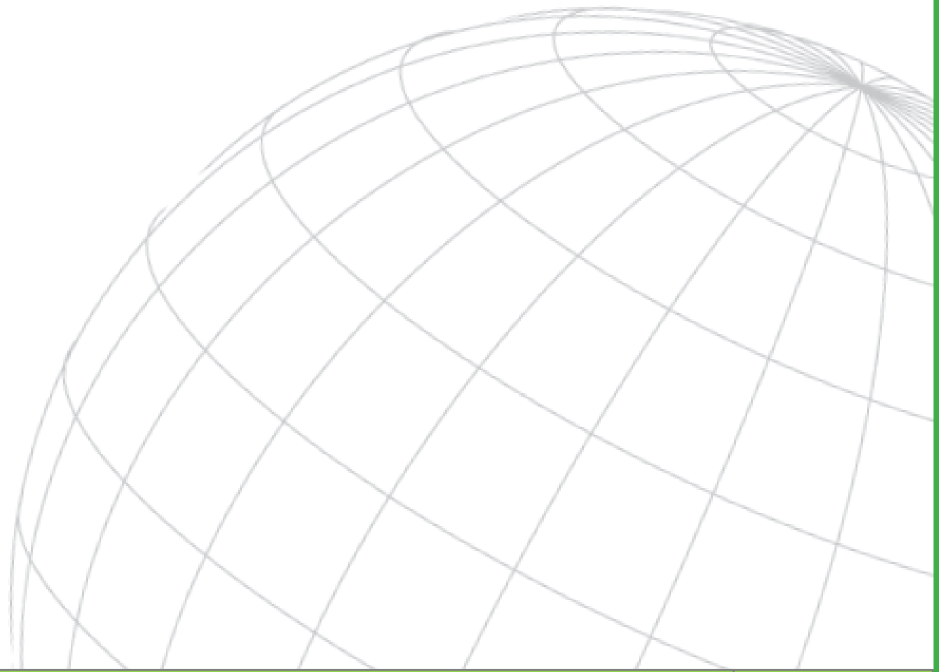
Obsah:

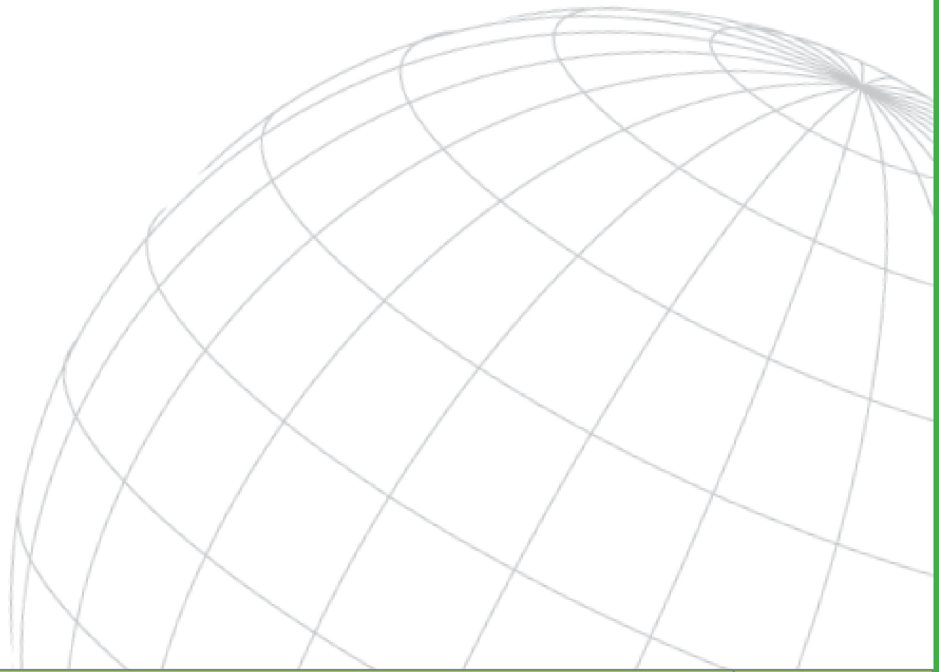
- Motivácia,
- Teoretický základ,
- Numerický experiment,
- Výsledky,
- Záver,



Motivácia:

- Družicová misia GOCE – viac ako 3 roky meraní a niekoľko druhov globálnych tiažových modelov Zeme,

- Numerické porovnanie troch metód na minimalizovanie vplyvu vzdialených zón.



Teoretický základ: Integrálne transformácie

Vzťah medzi prvkami poruchového gravitačného tenzora v LNOF a tiažovými poruchami je definovaný nasledujúcou integrálnou rovnicou (Pitoňák a kol., 2017):

$$T_{i,j}(r,\Omega)=\frac{R}{4\pi}\int_{\Omega'}\delta g(R,\Omega')H_{i,j}(t,\psi)\mathrm{d}\Omega',\ i,j=x,y,z\,, \tag{1}$$

kde príslušné integrálne jadrá sú definované ako (Wolf a Denker, 2005):

$$\begin{gathered}\left\{\begin{matrix}H_{xx}(t,\psi,\alpha)\\ H_{yy}(t,\psi,\alpha)\end{matrix}\right\}=\frac{1}{r}H_{r}(t,\psi)+\frac{1}{2r^{2}}\Big(\big[H_{\psi\psi}(t,\psi)+H_{\psi}(t,\psi)\mathrm{cotg}\,\psi\big]\pm\big[H_{\psi\psi}(t,\psi)-H_{\psi}(t,\psi)\mathrm{cotg}\,\psi\big]\cos 2\alpha\Big),\\ H_{zz}(t,\psi)=H_{rr}(t,\psi),\ H_{xz}(t,\psi,\alpha)=\frac{1}{r}\left[\frac{1}{r}H_{\psi}(t,\psi)-H_{\psi r}(t,\psi)\right]\cos\alpha.\end{gathered} \tag{2}$$

Príslušné derivácie Hotineho funkcie podľa ψ a r sú definované ako (Pitoňák a kol., 2017):

$$\begin{gathered}H_{\psi}(t,\psi)=\frac{\sin\psi}{\cos\psi+1}-\frac{\sin\psi}{D-t+\cos\psi}+\frac{t\sin\psi}{D\left(D-t+\cos\psi\right)}-\frac{2t^{2}\sin\psi}{D^{3}},\quad H_{r}(t,\psi)=\frac{t^{3}-t}{rD^{3}},\quad H_{\psi r}(t,\psi)=\frac{3t\sin\psi\left(t-t^{3}\right)}{rD^{5}},\quad H_{\psi r}(t,\psi)=\frac{3t\sin\psi\left(t-t^{3}\right)}{rD^{5}},\\ H_{\psi\psi}(t,\psi)=\frac{\cos\psi}{\cos\psi+1}-\frac{\left(D-t\right)^{2}\left(\cos 2\psi-1\right)}{2D^{2}\left(D-t+\cos\psi\right)^{2}}+\frac{\sin^{2}\psi}{\left(\cos\psi+1\right)^{2}}+\frac{6t^{3}\sin^{2}\psi}{D^{5}}-\frac{2t^{2}\cos\psi}{D^{3}}-\frac{D^{3}\cos\psi-D^{2}t\cos\psi-t^{2}\cos^{2}\psi+t^{2}}{D^{3}\left(D-t+\cos\psi\right)}.\end{gathered} \tag{3}$$

Teoretický základ: Inverzná úloha

Rovnica (1) vo vektorov tvare je známa ako Gauss-Markovov model:

$$\mathbf{l} = \mathbf{A}\mathbf{x} + \boldsymbol{\varepsilon}, \qquad (4)$$

pre odhad neznámeho parametra **x** pomocou MNŠ vieme napísať

$$\hat{\mathbf{x}} = \left(\mathbf{A}^T\mathbf{P}\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{P}\mathbf{l} = \mathbf{N}^{-1}\mathbf{A}^T\mathbf{P}\mathbf{l}. \qquad (5)$$

Systém normálnych rovníc (5) reprezentuje diskrétny tvar Fredholmovej integrálnej rovnice prvého druhu so zle podmienenou maticou **N** (jej číslo podmienenosti je ≈ 10^{16}). Na numerické stabilizovanie riešenia sme použili Tichonovovu regularizáciu (Tikhonov 1963a, b) definovanú ako:

$$\hat{\mathbf{x}}_{reg} = \left(\mathbf{A}^T\mathbf{P}\mathbf{A} + \alpha^2\mathbf{I}\right)^{-1}\mathbf{A}^T\mathbf{P}\mathbf{l} = \left(\mathbf{N} + \alpha^2\mathbf{I}\right)^{-1}\mathbf{A}^T\mathbf{P}\mathbf{l}. \qquad (6)$$

Regularizačný parameter α^2 bol určený pomocou metódy general cross-validation (GCV; Hansen and O'Fleary, 1993).

Kombinácia štyroch presne meraných gradientov:

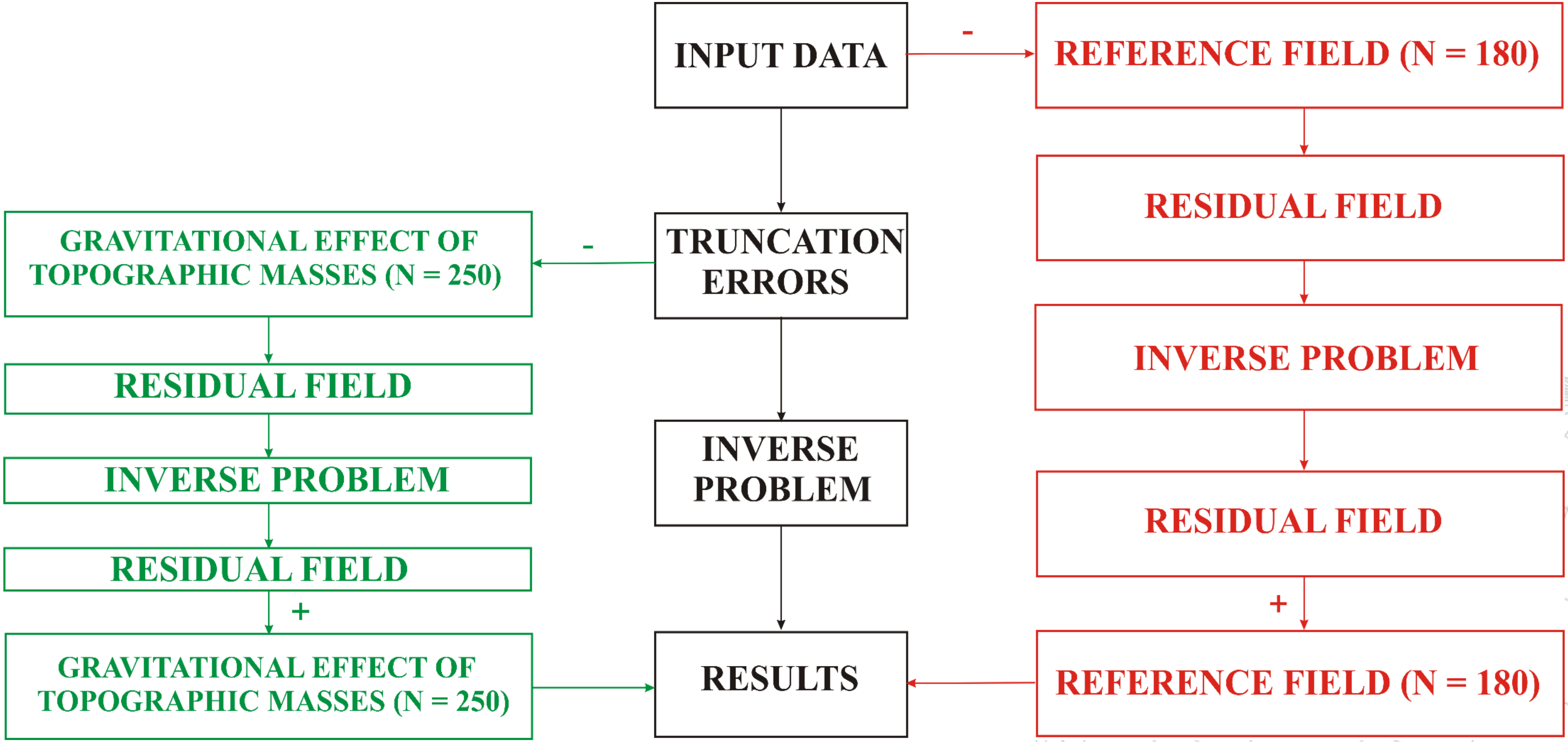
Tichonovova regularizácia (rovnica 6)

$$\hat{\mathbf{x}}_{reg} = \left(\mathbf{A}^T\mathbf{P}\mathbf{A} + \alpha^2\mathbf{I}\right)^{-1}\mathbf{A}^T\mathbf{P}\mathbf{l} = \left(\mathbf{N} + \alpha^2\mathbf{I}\right)^{-1}\mathbf{A}^T\mathbf{P}\mathbf{l}$$

$$\mathbf{l} = [\mathbf{l}_{xx} \quad \mathbf{l}_{yy} \quad \mathbf{l}_{zz} \quad \mathbf{l}_{xz}]^T$$

$$\mathbf{A} = [\mathbf{A}_{xx} \quad \mathbf{A}_{yy} \quad \mathbf{A}_{zz} \quad \mathbf{A}_{xz}]^T$$

Numerický experiment: Výpočtové schémy



INPUT DATA

TRUNCATION ERRORS

INVERSE PROBLEM

RESULTS

-

REFERENCE FIELD (N = 180)

RESIDUAL FIELD

INVERSE PROBLEM

RESIDUAL FIELD

+

REFERENCE FIELD (N = 180)

-

GRAVITATIONAL EFFECT OF TOPOGRAPHIC MASSES (N = 250)

RESIDUAL FIELD

INVERSE PROBLEM

RESIDUAL FIELD

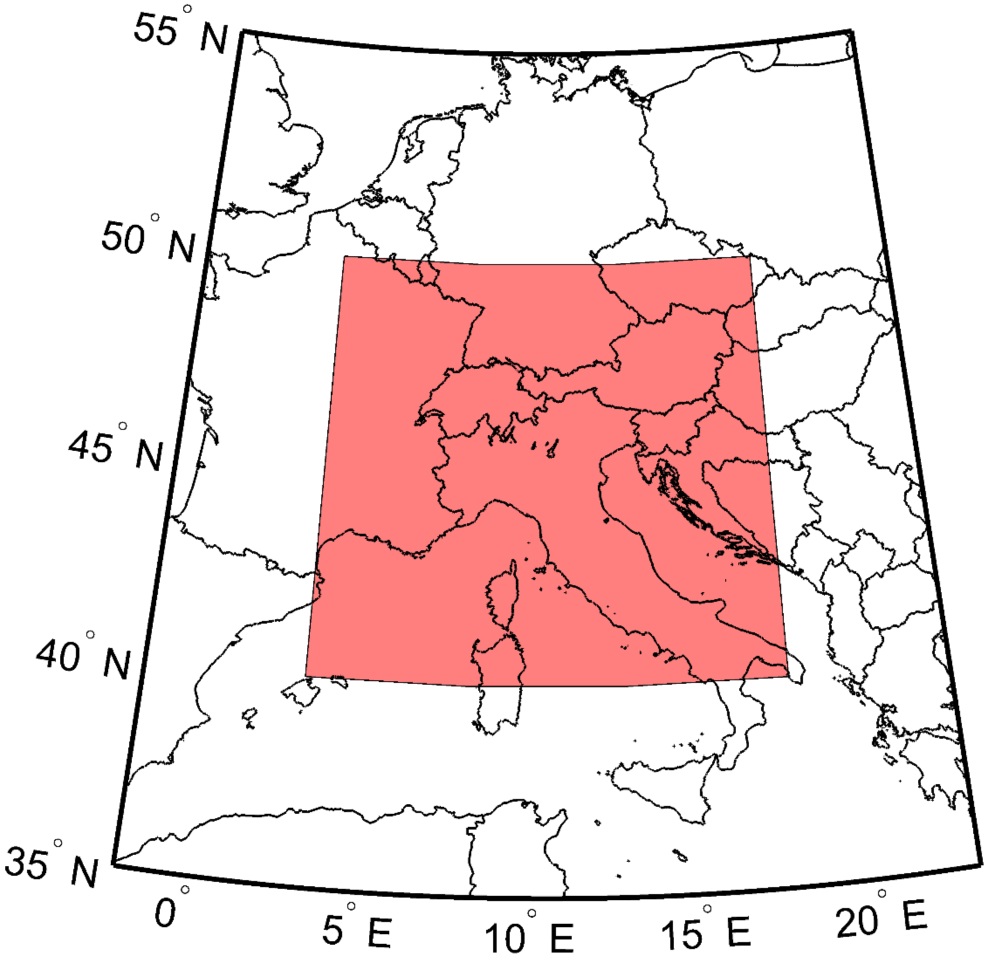
+

GRAVITATIONAL EFFECT OF TOPOGRAPHIC MASSES (N = 250)

Numerický experiment: Vstupné dáta

- EGG_TRF_2 (1. november 2009 – 11. január 2010) ≈ 2.5 months,
- Rock-Water-Ice gravitačný model izostaticky-kompenzovanej topografie (Grombein a kol. 2014),
- TIM-r4 (Pail a kol. 2011),
- EGM2008 (Pavlis et al. 2012).

Numerický experiment: Testovacia oblasť



- Testovacia oblasť:

  $\varphi \in [40°;50°],\ \lambda \in [3°;18°],\ \Delta\varphi = \Delta\lambda = 0.2°,\ R = 6388136.3\ \text{m}$

- oblasť pokrytia meraniami družice GOCE

  $\varphi \in [35°;55°],\ \lambda \in [-2°;23°]$

Výsledky: Rozdiely medzi našimi riešeniami a EGM2008 d/o 250 (klasická metóda)

	T_{xx}	T_{yy}	T_{zz}	T_{xz}	kombinácia
STD	8,2	8,7	7,9	9,8	**6,9**
MIN	-27,5	-33,1	-31,0	-35,3	-28,1
MAX	23,8	35,4	21,4	42,6	25,2
MEAN	-0,2	-0,8	-0,0	-1,1	-0,2

(jednotky mGal)

Výsledky: Rozdiely medzi našimi riešeniami a EGM2008 d/o 250 (metóda R-C-R s RWI modelom)

	T_{xx}	T_{yy}	T_{zz}	T_{xz}	kombinácia
STD	6,2	8,3	**4,4**	6,7	7,9
MIN	-20,5	-34,0	-19,5	-24,0	-27,8
MAX	22,8	22,1	15,3	19,8	43,5
MEAN	0,4	-0,9	0,2	-1,4	0,1

(jednotky mGal)

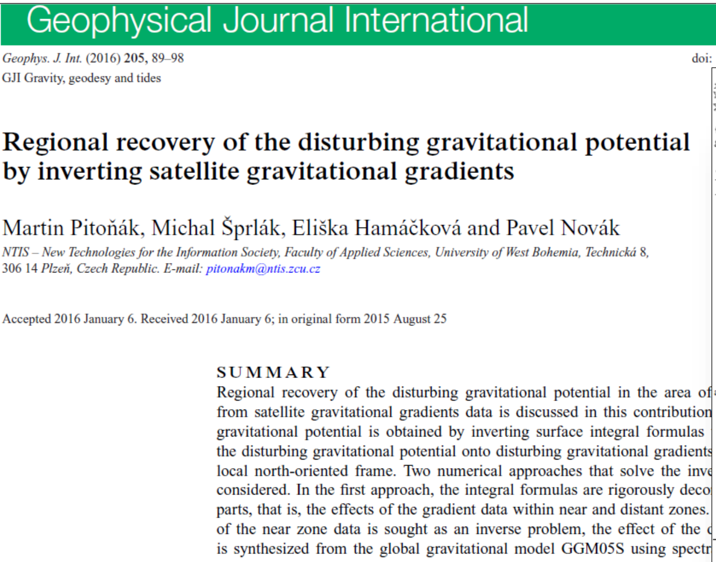
Výsledky: Rozdiely medzi našimi riešeniami a EGM2008 d/o 250 (metóda R-C-R)

	T_{xx}	T_{yy}	T_{zz}	T_{xz}	kombinácia
STD	8,0	7,2	**5,7**	9,4	6,2
MIN	-23,2	-34,9	-29,5	-32,5	-31,2
MAX	26,2	27,0	21,2	37,7	23,9
MEAN	0,1	-0,2	0,1	-0,2	0,0

(jednotky mGal)

Záver:

- Tri metódy na minimalizovanie vplyvu vzdialených zón boli porovnané,
- Z testovaných metód boli najpresnejšie výsledky dosiahnuté pomocou metódy R-C-R s RWI modelom,
- Najlepšie výsledky pri použití jednotlivých prvkov poruchového gravitačného tenzora boli dosiahnuté z komponentu T_{zz}, ktorý má na dráhe družice najsilnejší signál,
- Pri kombinovaní všetkých štyroch komponentov bola najlepšia zhoda s modelom EGM2008 dosiahnutá pomocou metódy R-C-R



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GJI Gravity, geodesy and tides

Regional recovery of the disturbing gravitational potential by inverting satellite gravitational gradients

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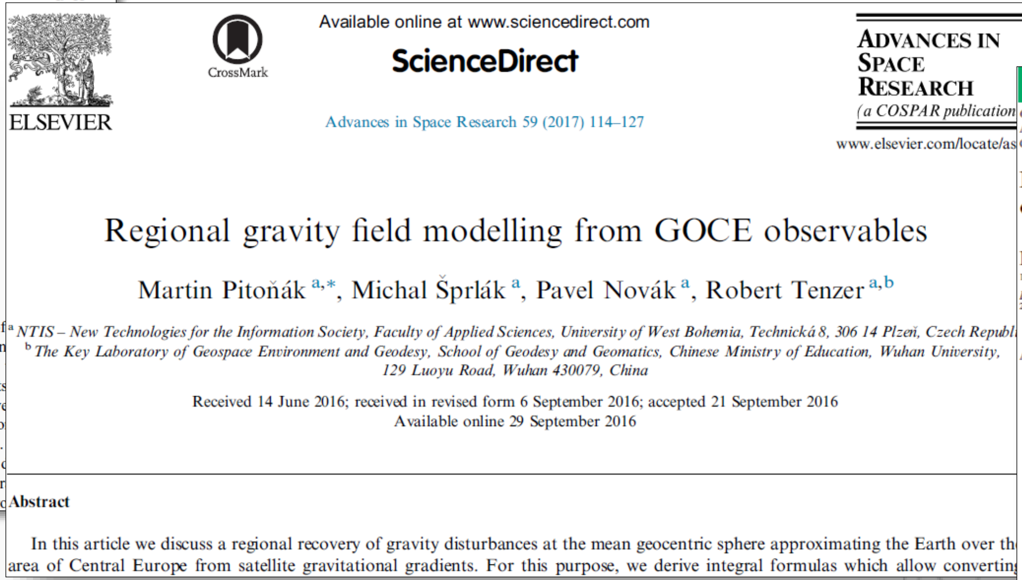
SUMMARY
Regional recovery of the disturbing gravitational potential in the area of
from satellite gravitational gradients data is discussed in this contribution
gravitational potential is obtained by inverting surface integral formulas
the disturbing gravitational potential onto disturbing gravitational gradients
local north-oriented frame. Two numerical approaches that solve the inve
considered. In the first approach, the integral formulas are rigorously deco
parts, that is, the effects of the gradient data within near and distant zones.
of the near zone data is sought as an inverse problem, the effect of the
is synthesized from the global gravitational model GGM05S using spectr
by truncation error coefficients up to the degree 150. In the second appro



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Regional gravity field modelling from GOCE observables

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Abstract

In this article we discuss a regional recovery of gravity disturbances at the mean geocentric sphere approximating the Earth over th
area of Central Europe from satellite gravitational gradients. For this purpose, we derive integral formulas which allow convertin

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GJI Gravity, geodesy and tides

Possibilities of inversion of satellite third-order gravitational tensor onto gravity anomalies: a case study for central Europe

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SUMMARY
We investigate a numerical performance of four different schemes applied to a regional recovery of the gravity anomalies from the third-order gravitational tensor components (assumed to be observable in the future) synthetized at the satellite altitude of 200 km above the mean sphere. The first approach is based on applying a regional inversion without modelling the far-zone contribution or long-wavelength support. In the second approach we separate integral formulas into two parts, that is, the effects of the third-order disturbing tensor data within near and far zones. Whereas the far-zone contribution is evaluated by using existing global geopotential model (GGM) with spectral weights given by truncation error coefficients, the near-zone contribution is solved by applying a regional inversion. We then extend this approach for a smoothing procedure, in which we remove the gravitational contributions of the topographic-isostatic and atmospheric masses. Finally, we apply the remove-compute-restore (r-c-r) scheme in order to reduce the far-zone contribution by subtracting the reference (long-

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